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## LETTER TO THE EDITOR

## Absorbing phase transition in a conserved lattice gas with random neighbour particle hopping

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### Abstract

A conserved lattice gas with random neighbour hopping of active particles is introduced which exhibits a continuous phase transition from an active state to an absorbing non-active state. Since the randomness of the particle hopping breaks long-range spatial correlations our model mimics the mean-field scaling behaviour of the recently introduced new universality class of absorbing phase transitions with a conserved field. The critical exponents of the order parameter are derived within a simple approximation. The results are compared with those of simulations and field theoretical approaches.

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Recently Rossi *et al* introduced a conserved lattice gas (CLG) with a stochastic short-range interaction that exhibits a continuous phase transition to an absorbing state at a critical value of the particle density [1]. The CLG model is expected to belong to the universality class of absorbing phase transitions with a conserved field [1]. This universality class is different from the well known universality class of directed percolation (see [2, 3] or for an overview [4]). In this work we introduce a modified CLG model with a random neighbour interaction. This random neighbour interaction suppresses long-range correlations and the model is expected to be characterized by the mean-field scaling behaviour. The critical exponent of the absorbing phase transition is derived within a simple approximation. Numerical simulations of various lattice types with different numbers of next neighbours confirm the obtained results.

Let us consider a system consisting of  $L$  sites on a chain with periodic boundary conditions. At the beginning one distributes randomly  $N = \rho L$  particles on the system where  $\rho$  denotes the particle density. A particle is called active if at least one of its two neighbouring sites is occupied. In the original CLG model active particles jump in the next update step to one of their empty nearest-neighbour sites, selected at random [1]. In the steady state the system is characterized by the density of active sites  $\rho_a$  which depends on  $\rho$ . The density of inactive sites is given by  $\rho - \rho_a$  and  $1 - \rho$  is the density of empty sites. The density  $\rho_a$  is the order

parameter of the absorbing phase transition [1], i.e. it vanishes at the so-called critical density of particles  $\rho_c$  (which, of course, is lower than the trivial value  $\rho = 1/2$ ).

In our modification of the CLG model active particles are moved to a randomly chosen empty lattice site which suppresses long-range correlations. On the other hand, short-range correlations still do exist. For instance, close to the critical point, where the density of active sites is sufficiently low, the density–density correlation function displays an alternating behaviour due to the repulsive interaction of the particles. But these correlations are short ranged and it is therefore possible to neglect them. The values of the critical exponents are not affected by this approximation.

Let us consider a configuration  $\mathcal{C}$  of the lattice with  $n$  active particles. The number of active sites may change in each particle hopping, i.e. in each elementary update step. For instance, if both new neighbours of the hopped particle are empty the number of active particles is reduced by one,  $\Delta n = -1$ . Without correlations the corresponding probability is  $(1 - \rho)^2$ . If one of the new neighbours of the hopped particle is occupied by an inactive particle ( $p = \rho - \rho_a$ ) and the second neighbour is empty ( $p = 1 - \rho$ ), the number of active sites is increased by one ( $\Delta n = 1$ ), and the corresponding probability is given by  $p = 2(1 - \rho)(\rho - \rho_a)$ . All other possible configurations and the corresponding probabilities are listed in table 1.

**Table 1.** The configuration of the lattice before ( $\mathcal{C}$ ) and after ( $\mathcal{C}'$ ) a particle hopping. Only the target lattice site where a particle hops and its left and right neighbouring sites are shown. Empty sites are marked by  $\circ$ , inactive sites are marked by  $*$ , and active sites by  $\bullet$ .  $\Delta n$  denotes the change of the number of active sites due to the particle hopping and  $p$  is the corresponding probability of the configuration  $\mathcal{C}$  if one neglects spatial correlations.

$\mathcal{C}$	$\mathcal{C}'$	$\Delta n$	$p(\mathcal{C})$
$\circ \circ \circ$	$\circ * \circ$	-1	$(1 - \rho)^2$
$* \circ \circ$	$\bullet \bullet \circ$	+1	$2(1 - \rho)(\rho - \rho_a)$
$* \circ *$	$\bullet \bullet \bullet$	+2	$(\rho - \rho_a)^2$
$\bullet \circ \circ$	$\bullet \bullet \circ$	0	$2\rho_a(1 - \rho)$
$\bullet \circ \bullet$	$\bullet \bullet \bullet$	0	$\rho_a^2$
$\bullet \circ *$	$\bullet \bullet \bullet$	+1	$2\rho_a(\rho - \rho_a)$

In this way one can calculate the probabilities that the number of active particles are changed by  $\Delta n$  and one gets

$$\begin{aligned}
 p_{\Delta n=-1} &= (1 - \rho)^2 \\
 p_{\Delta n=0} &= 2\rho_a(1 - \rho) + \rho_a^2 \\
 p_{\Delta n=1} &= 2(\rho - \rho_a)(1 - \rho) + 2\rho_a(\rho - \rho_a) \\
 p_{\Delta n=2} &= (\rho - \rho_a)^2.
 \end{aligned}
 \tag{1}$$

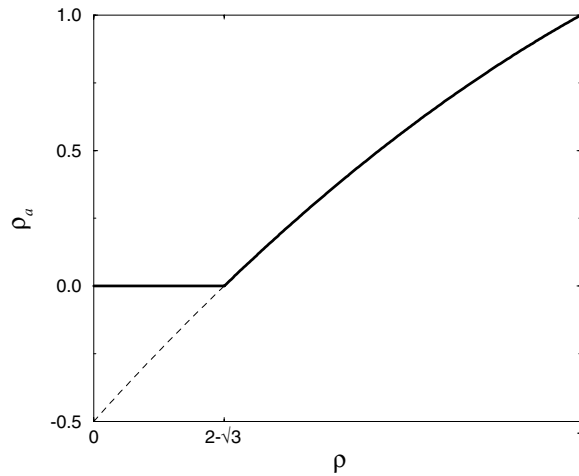
The expectation value of  $\Delta n$  is

$$E[\Delta n] = \sum_{\Delta n=-1}^2 \Delta n p_{\Delta n} = -1 - 2\rho_a + 4\rho - \rho^2.
 \tag{2}$$

The average number of active sites is constant in the stationary state, i.e. the expectation value of  $\Delta n$  should be zero in the steady state. Using the constraint  $E[\Delta n] = 0$  it is possible to calculate  $\rho_a$  as a function of  $\rho$  and one gets

$$\rho_a = \frac{4\rho - \rho^2 - 1}{2}.
 \tag{3}$$

The corresponding curve is plotted in figure 1 for  $0 < \rho < 1$ . Negative values of  $\rho_a$  corresponds to an absorbing state (i.e.  $\rho_a = 0$ ). The critical point is determined by  $\rho_a = 0$ , which leads to



**Figure 1.** The density of active sites  $\rho_a$  as a function of the particle density  $\rho$  in the steady state. Negative values (dashed curve) correspond to an absorbing state with  $\rho_a = 0$ .

$\rho_c = 2 - \sqrt{3}$  (the second solution can be neglected since  $\rho_c = 2 + \sqrt{3} > 1$ ). Writing  $\rho_a$  as a function of the reduced density  $\delta\rho = \rho - \rho_c$  one gets

$$\rho_a = \sqrt{3} \delta\rho - \frac{1}{2} \delta\rho^2. \tag{4}$$

Thus the order parameter of the absorbing phase transition vanishes in leading order as  $\rho_a \sim \delta\rho$ , i.e. the order parameter exponent is  $\beta = 1$ .

We briefly remark that it is straightforward to generalize the above derivation from a chain with two neighbours to a  $d$ -dimensional cubic lattice with  $z = 2d$  neighbours. Since  $\Delta n$  depends on the number of inactive sites (\*) only (see table 1) the corresponding probabilities are just polynomials in  $\rho_* = \rho - \rho_a$ , i.e.

$$\begin{aligned} p_{\Delta n=-1} &= (1 - \rho)^z \\ p_{\Delta n=0} &= (1 - \rho_*)^z - (1 - \rho)^z \\ p_{\Delta n \geq 1} &= \binom{z}{\Delta n} \rho_*^{\Delta n} (1 - \rho_*)^{z-\Delta n}. \end{aligned} \tag{5}$$

Using again the steady state condition  $E[\Delta n] = 0$  one gets

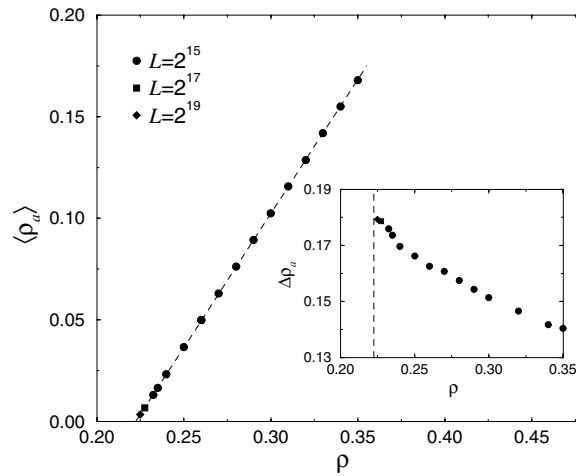
$$\rho_a = \rho - \frac{(1 - \rho)^z}{z}. \tag{6}$$

The critical density  $\rho_c$  is determined by  $\rho_a = 0$ . Expanding equation (6) around  $\rho_c$  yields

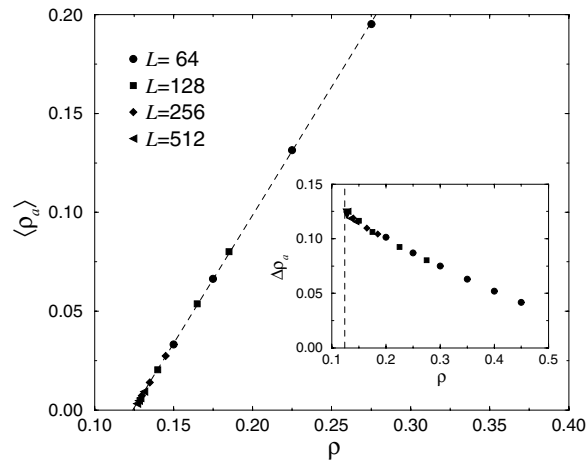
$$\rho_a = (2 - \rho_c) \delta\rho + \mathcal{O}(\delta\rho^2), \tag{7}$$

i.e. the order parameter exponent is  $\beta = 1$  independent of the number of next neighbours.

In the following we compare these results with those obtained from simulations. We simulated a one-dimensional chain ( $z = 2$ ) and a two-dimensional square lattice ( $z = 4$ ) where the active particles are moved to a randomly chosen empty lattice site. In both cases a random sequential update was used and the results are plotted in figures 2 and 3, respectively. As expected, the critical density  $\rho_c$  decreases with the number of next neighbours  $z$ . Note that the critical value of the chain  $\rho_c \approx 0.2224$  and of the square lattice  $\rho_c \approx 0.1244$  differs slightly from the above analytical result  $\rho_c = 0.2679\dots$  and  $\rho_c = 0.1380\dots$ , respectively.



**Figure 2.** The average density of active sites for a one-dimensional chain of size  $L$  with random particle hopping ( $z = 2$ ). The dashed line corresponds to a linear fit, i.e. the order parameter exponent is  $\beta = 1$ . The inset displays the fluctuations of the order parameter which exhibits a discontinuous behaviour at the critical density  $\rho_c$  (dashed line).



**Figure 3.** The density of active sites for a two-dimensional square lattice of linear size  $L$  with random particle hopping ( $z = 4$ ). The dashed line corresponds to a linear fit, i.e. the order parameter exponent is  $\beta = 1$ . The inset displays the fluctuations of the order parameter which exhibits a discontinuous behaviour at the critical density  $\rho_c$  (dashed line).

This deviation, which decreases with increasing  $z$ , is caused by the neglect of correlations between neighbouring sites. Simulations reveal that these correlations exist but are of short range (not shown). Therefore, the critical value is shifted but the scaling behaviour itself agrees with our analytical results, i.e. the order parameter vanishes at the critical point continuously with an exponent  $\beta = 1$ .

Additionally to the order parameter, its fluctuations

$$\Delta \rho_a = L^D (\langle \rho_a^2 \rangle - \langle \rho_a \rangle^2) \quad (8)$$

are measured in the simulations. Here  $D$  denotes the dimension of the system. As seen in the insets of figures 2 and 3, the fluctuations exhibit a discontinuous behaviour (jump) at  $\rho_c$ .

The value  $\beta = 1$  and the jump of the fluctuations was also observed in the CLG model above the critical dimension  $D_c = 4$  [5] where the scaling behaviour of the model is determined by the mean-field exponents. Furthermore, the values  $\beta = 1$  and  $D_c = 4$  were predicted within a field theoretical approach which is expected to represent the universality class of absorbing phase transitions with a conserved field [6, 7].

Finally, we just mention that the derivation of our results corresponds to a mapping of the dynamics to a branching process (see, for instance, [8]). There each active site can create  $i \in \{0, 1, 2, 3\}$  active sites in the next generation with the probability  $p_i = p_{\Delta n=i-1}$ . The steady state condition corresponds to the condition that the average number of created active sites in the next generation is one, which yields directly equation (3).

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